Using the distance based definition of a hyperbola, find the equation of the hyperbola with foci $(0, \pm 12)$ such that the distances from any point on the hyperbola to the foci differ by 8. **SCORE: _____ / 10 PTS**

$$\left|\sqrt{x^2+(y+12)^2}-\sqrt{x^2+(y-12)^2}\right|=8$$

$$\boxed{\sqrt{x^2+(y+12)^2}-\sqrt{x^2+(y-12)^2}=\pm 8} \quad (2)$$

EARN ONLY 1 POINT IF YOU WROTE 8 INSTEAD OF ±8

$$(1)\ \sqrt{x^2+(y+12)^2}=\pm 8+\sqrt{x^2+(y-12)^2}$$

$$\boxed{x^2+y^2+24y+144}=\boxed{64\pm 16\sqrt{x^2+y^2-24y+144}+x^2+y^2-24y+144} \quad (2)$$

$$48y-64=\pm 16\sqrt{x^2+y^2-24y+144}$$

$$\boxed{3y-4=\pm\sqrt{x^2+y^2-24y+144}} \quad (2)$$

$$\boxed{9y^2-24y+16=x^2+y^2-24y+144} \quad (1)$$

$$\boxed{8y^2-x^2=128} \quad (1)$$

$$\boxed{\frac{y^2}{16}-\frac{x^2}{128}=1} \quad (1)$$

ALTERNATE SOLUTIONS ON PAGE 3

Convert the rectangular equation $y^2=x^2-5$ to polar form. Write r as function of θ, and simplify your answer. **SCORE: _____ / 4 PTS**

$$(r\sin\theta)^2=(r\cos\theta)^2-5$$

$$\boxed{r^2\sin^2\theta=r^2\cos^2\theta-5} \quad (1)$$

$$5=r^2\cos^2\theta-r^2\sin^2\theta$$

$$\boxed{5=r^2(\cos^2\theta-\sin^2\theta)} \quad (1)$$

$$\boxed{\frac{5}{\cos^2\theta-\sin^2\theta}=r^2} \quad (1)$$

$$\boxed{\frac{5}{\cos 2\theta}=r^2} \quad \left(\tfrac{1}{2}\right)$$

$$\boxed{r^2=5\sec 2\theta} \quad \left(\tfrac{1}{2}\right)$$

Fill in the blanks. **SCORE: _____ / 7 PTS**

[a] A house has an exposed (straight) beam 25 feet above and parallel to the floor. A small lamp hangs from the ceiling.

There is an arch such that the distance from any point on the arch to the lamp is the same as the distance from that point to the beam.

The shape of the arch is a/an PARABOLA (or part of it). (2)

[b] The shape of the graph of the equation $3x^2 - 3x + 2y^2 - 2y - 1 = 0$ is a/an ELLIPSE. (1)

[c] The shape of the graph of the equation $3x^2 + 2x + 3y^2 - 3y - 1 = 0$ is a/an CIRCLE. (1)

[d] The polar co-ordinates $(-5, \frac{4\pi}{7})$ refer to the same point as the polar co-ordinates $(5, \frac{11\pi}{7})$. (1) (Your answer must be **positive**.)

[e] The polar co-ordinates $(5, \frac{4\pi}{7})$ refer to the same point as the polar co-ordinates $(5, \frac{-10\pi}{7})$. (1) (Your answer must be **negative**.)

[f] The point with polar co-ordinates $(-5, -\frac{2\pi}{3})$ lies in quadrant I. (1)

Convert the polar equation $r^2 = \cos 2\theta$ to rectangular form. **SCORE: _____ / 4 PTS**
Simplify your answer so that there are no radicals, complex fractions, fractional exponents nor negative exponents.

$r^2 = \cos^2\theta - \sin^2\theta$ (1)

$r^2 = \left(\frac{x}{r}\right)^2 - \left(\frac{y}{r}\right)^2$ OR $r^2 r^2 = r^2(\cos^2\theta - \sin^2\theta)$ (1) EITHER VERSION OK

$r^2 = \frac{x^2}{r^2} - \frac{y^2}{r^2}$ OR $r^4 = r^2\cos^2\theta - r^2\sin^2\theta$

$r^4 = x^2 - y^2$ (1)

$(x^2 + y^2)^2 = x^2 - y^2$ (1)

Find the vertices, foci and equations of the asymptotes of the hyperbola $x^2 - 2y^2 - 4x - 16y - 22 = 0$. **SCORE: _____ / 5 PTS**

$(x^2 - 4x) - 2(y^2 + 8y) = 22$

$(x^2 - 4x + 4) - 2(y^2 + 8y + 16) = 22 + 4 - 32$ $\left(\frac{1}{2}\right)$

$(x-2)^2 - 2(y+4)^2 = -6$ $\left(\frac{1}{2}\right)$

$\frac{(y+4)^2}{3} - \frac{(x-2)^2}{6} = 1$ $\left(\frac{1}{2}\right)$

VERTICES: $(2, -4 \pm \sqrt{3})$ $\left(\frac{1}{2}\right)$

FOCI: $(2, -4 \pm 3) = (2, -7), (2, -1)$ (1)

ASYMPTOTES: $y + 4 = \pm\frac{\sqrt{2}}{2}(x - 2)$ (1) (1)

$$\left|\sqrt{x^2+(y-12)^2}-\sqrt{x^2+(y+12)^2}\right|=8$$

ALTERNATE SOLUTION 1

$$\sqrt{x^2+(y-12)^2}-\sqrt{x^2+(y+12)^2}=\pm 8$$ (2) ← EARN ONLY 1 POINT IF YOU WROTE 8 INSTEAD OF ±8

$$\sqrt{x^2+(y-12)^2}=\pm 8+\sqrt{x^2+(y+12)^2}$$

(1) $$x^2+y^2-24y+144=64\pm 16\sqrt{x^2+y^2+24y+144}+x^2+y^2+24y+144$$ (2)

$$-48y-64=\pm 16\sqrt{x^2+y^2+24y+144}$$

$$-3y-4=\pm\sqrt{x^2+y^2+24y+144}$$ (2)

$$9y^2+24y+16=x^2+y^2+24y+144$$ (1)

$$8y^2-x^2=128$$ (1)

$$\frac{y^2}{16}-\frac{x^2}{128}=1$$ (1)

ALTERNATE SOLUTION 2

EARN ONLY 1 POINT IF YOU WROTE 2a INSTEAD OF ±2a

$$\left|\sqrt{x^2+(y+c)^2}-\sqrt{x^2+(y-c)^2}\right|=2a$$

$$\sqrt{x^2+(y+c)^2}-\sqrt{x^2+(y-c)^2}=\pm 2a$$ (2)

$$\sqrt{x^2+(y+c)^2}=\pm 2a+\sqrt{x^2+(y-c)^2}$$

(1) $$x^2+y^2+2cy+c^2=4a^2\pm 4a\sqrt{x^2+y^2+2cy+c^2}+x^2+y^2-2cy+c^2$$ (2)

$$4cy-4a^2=\pm 4a\sqrt{x^2+y^2-2cy+c^2}$$

$$cy-a^2=\pm a\sqrt{x^2+y^2-2cy+c^2}$$ (2)

$$c^2y^2-2a^2cy+a^4=a^2(x^2+y^2-2cy+c^2)$$ (1)

$$c^2y^2-2a^2cy+a^4=a^2x^2+a^2y^2-2a^2cy+a^2c^2$$

$$c^2y^2-a^2y^2-a^2x^2=a^2c^2-a^4$$

$$(c^2-a^2)y^2-a^2x^2=a^2(c^2-a^2)$$ (1)

$$\frac{y^2}{a^2}-\frac{x^2}{c^2-a^2}=1$$

$$\frac{y^2}{16}-\frac{x^2}{128}=1$$ (1)

$$\left|\sqrt{x^2+(y-c)^2}-\sqrt{x^2+(y+c)^2}\right|=2a$$

ALTERNATE SOLUTION 3

$$\sqrt{x^2+(y-c)^2}-\sqrt{x^2+(y+c)^2}=\pm 2a$$ (2) ← EARN ONLY 1 POINT IF YOU WROTE 2a INSTEAD OF ±2a

$$\sqrt{x^2+(y-c)^2}=\pm 2a+\sqrt{x^2+(y+c)^2}$$

(1) $$x^2+y^2-2cy+c^2=4a^2\pm 4a\sqrt{x^2+y^2+2cy+c^2}+x^2+y^2+2cy+c^2$$ (2)

$$-4cy-4a^2=\pm 4a\sqrt{x^2+y^2+2cy+c^2}$$

$$-cy-a^2=\pm a\sqrt{x^2+y^2+2cy+c^2}$$ (2)

$$c^2y^2+2a^2cy+a^4=a^2(x^2+y^2+2cy+c^2)$$ (1)

$$c^2y^2+2a^2cy+a^4=a^2x^2+a^2y^2+2a^2cy+a^2c^2$$

$$c^2y^2-a^2y^2-a^2x^2=a^2c^2-a^4$$

$$(c^2-a^2)y^2-a^2x^2=a^2(c^2-a^2)$$ (1)

$$\frac{y^2}{a^2}-\frac{x^2}{c^2-a^2}=1$$

$$\frac{y^2}{16}-\frac{x^2}{128}=1$$ (1)